



DQ-003-2016002

Seat No. _____

B. Sc. (Sem. VI) (CBCS) Examination

April - 2022

Mathematics : Paper - M - 09 (A)

**(Mathematical Analysis - II & Abstract Algebra - II)
(New Course)**

Faculty Code : 003

Subject Code : 2016002

Time : $2\frac{1}{2}$ Hours]

[Total Marks : 70

Instructions : (1) All questions are compulsory.
(2) Write answer of each question in your main answer sheet.

- 1 (a) Answer the following questions briefly : 4
- (1) Define Open Cover.
 - (2) Define : Disconnected set.
 - (3) Determine whether the subset $\{2, 7\}$ of metric space R is compact or not.
 - (4) Define compact set.
- (b) Attempt any one out of two : 2
- (1) Show that subset $R - \{a\}$ is not connected, where $a \in R$.
 - (2) If A and B are compact sets of metric space X then prove that $A \cap B$ is also compact.
- (c) Attempt any one out of two : 3
- (1) State and prove Heine-Borel theorem.
 - (2) If F is a closed subset of metric space X and K is a compact subset of X . Then prove that $F \cap K$ is also compact.
- (d) Attempt any one out of two : 5
- (1) State and prove theorem of nested intervals.
 - (2) Prove that continuous image of compact set is compact.

- 2 (a) Answer the following questions briefly : 4
- (1) Define Laplace transform.
 - (2) Find $L^{-1}\left(\frac{1}{s-1}\right)$
 - (3) Find $L^{-1}\left(\frac{1}{s^2-9}\right)$
 - (4) Show that $L(1)=\frac{1}{s}$, where $s > 0$.
- (b) Attempt any **one** out of two : 2
- (1) Find $L^{-1}\left(\frac{2s+6}{s^2+4}\right)$
 - (2) Find $L\left(\frac{e^{at}-1}{a}\right)$, where a is constant.
- (c) Attempt any one out of two : 3
- (1) Find Laplace transform of $e^{-2t} \sin^2 t$.
 - (2) If $L\{f(t)\} = \bar{f}(s)$ then prove that

$$L\{e^{at} f(t)\} = \bar{f}(s-a)$$
- (d) Attempt any one out of two : 5
- (1) If $f(t) = e^t$, $t \leq 2$
 $= 3$, $t > 2$ then find $L\{f(t)\}$.
 - (2) Prove that $L^{-1}\left(\frac{s^3}{s^4-a^4}\right) = \frac{1}{2}(\cos at + \cosh at)$
- 3 (a) Answer the following questions briefly : 4
- (1) Find $L(t \sin t)$
 - (2) Write convolution theorem.
 - (3) Find $L(t \sinh at)$
 - (4) Find $L\left(\frac{\sin t}{t}\right)$

(b) Attempt any one out of two : 2

(1) If $L\{f(t)\} = \bar{f}(s)$ then prove

$$L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} [\bar{f}(s)]$$

(2) If $L\{f(t)\} = \bar{f}(s)$ then prove $L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty \bar{f}(s) ds$.

(c) Attempt any one out of two : 3

(1) Prove that $L\{te^{-t} \sin t\} = \left(\frac{2(s+1)}{(s^2 + 2s + 2)^2} \right)$

(2) Prove that $L^{-1}\left(\log\left(\frac{s+b}{s+a}\right)\right) = \frac{e^{-at} - e^{-bt}}{t}$

(d) Attempt any one out of two : 5

(1) Prove that $L^{-1}\left\{\frac{1}{(s^2 + a^2)^2}\right\} = \frac{1}{2a^3}(\sin at - at \cos at)$

(2) Using convolution theorem, prove

$$L^{-1}\left\{\frac{1}{s(s^2 + 4)}\right\} = \frac{1}{4}(1 - \cos 2t)$$

4 (a) Answer the following questions briefly : 4

- (1) Define Ring
- (2) Define Homomorphism of Groups
- (3) Define Natural Mapping
- (4) Define Kernel of homomorphism

(b) Attempt any one out of two : 2

(1) Let $\phi: (G, *) \rightarrow (G', \Delta)$ is Homomorphism. If $H \leq G$ then prove $\phi(H) \leq G'$.

(2) If $\phi: (G, *) \rightarrow (G', \Delta)$ is a Homomorphism. Then $\phi(e) = e'$ where e and e' are identity elements of G and G' respectively.

- (c) Attempt any one out of two : 3
- (1) Prove that a cyclic group of order eight is homomorphism to a cyclic group of order four.
 - (2) If G is a cyclic group of prime order then prove that a homomorphism.
 $\phi: G \rightarrow G$ is either an isomorphism or
 $\phi(a) = e; \forall a \in G$
- (d) Attempt any one out of two : 5
- (1) State and prove first fundamental theorem of homomorphism.
 - (2) Prove that A homomorphism $\phi: (G, *) \rightarrow (G', \Delta)$ is one-of iff $\ker \phi = \{e\}$.
- 5 (a) Answer the following questions briefly : 4
- (1) Define constant polynomial.
 - (2) If polynomial $f = \{0, 3, 2, 7, 0, 0, 0, 0, \dots\}$ then find order of f .
 - (3) Define Linear polynomial.
 - (4) Define Monic polynomial.
- (b) Attempt any one out of two : 2
- (1) Find conjugate of quaternion $1 - 3i + 2j - k$.
 - (2) If $f(x) = (1, 3, 2, 2, 0, 0, \dots)$ and
 $g(x) = (2, 2, 0, 0, 3, 0, \dots) \in R[x]$ then find $f(x) + g(x)$.
- (c) Attempt any one out of two : 3
- (1) State and prove Remainder theorem of polynomials.
 - (2) Find g.c.d. of $f(x) = 6x^3 + 5x^2 - 2x + 25$ and
 $g(x) = 2x^2 - 3x + 5 \in R[X]$ and express it in the form
 $a(x)f(x) + b(x)g(x)$.
- (d) Attempt any one out of two : 5
- (1) State and prove division algorithm for polynomials.
 - (2) Prove that any ideal in integration domain $F[X]$ is a principal ideal.